Condensed matter physics of fractons

Rahul Nandkishore

University of Colorado Boulder

Department of Physics and Center for Theory of Quantum Matter

Prem, Haah, Nandkishore, Phys. Rev. B 95, 155133 (2017)

Prem, Pretko, Nandkishore, arXiv: 1709.09673

FQXi

Air Force Office of Scientific Research
United States Air Force

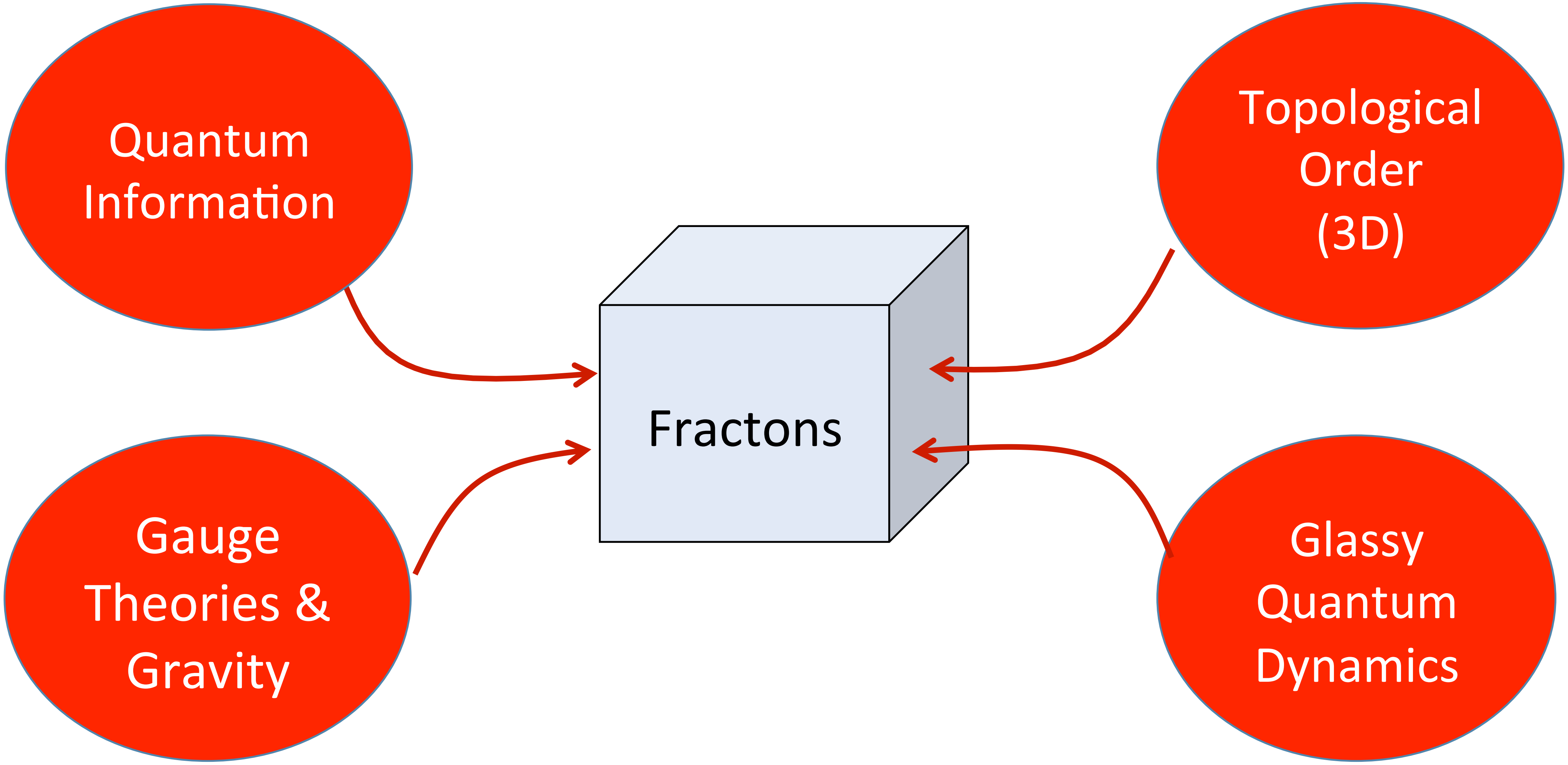
Alfred P. Sloan Foundation

What are fractons?

- New models of topological order, originally in three spatial dimensions
- Log(Ground state degeneracy) linear in system size, ground states indistinguishable by local measurement
- Immobile excitations (cannot be moved by any local operator). Sometimes also subdimensional excitations

Lightning history lesson

- Prehistory: Chamon 2005, Bravyi 2011
- **Haah 2011**
- Yoshida, various quantum information people…2011-2015
- Vijay-Haah-Fu (2015, 2016), Pretko 2016
- Prem-Haah-Nandkishore (2017), Ma-Lake-Chen-Hermele (2017), Vijay (2017), Pretko (2017), Hsieh-Halasz (2017), Slagle-(Y.B.)Kim (2017), Prem-Pretko-Nandkishore (2017), Ma-Schmitz-Parameswaran-Hermele-Nandkishore (2017), Devakul-Parameswaran-Sondhi (2017), He-Zheng-Bernevig-Regnault (2017)…



Ideas from all these fields inform study of fractons, and insight from fractons may inform all these fields

From fracton particle physics to fracton condensed matter

- Most work to date: studying ground states (fractonic vacuum) or few fracton problems (particle physics of fractons)
- My interest: fractons at finite density *(condensed matter physics of fractons)*
- Two directions
  - Discrete fracton models at finite *energy* density
  - Continuous fracton models at finite *charge* density

Fracton dynamics at non-zero energy density

Prem, Haah, Nandkishore, Phys. Rev. B 95, 155133 (2017)

Work with familiar models (X-cube, Haah's code)

Language of *perturbed stabilizer codes,* augmented by concepts from MBL/ETH

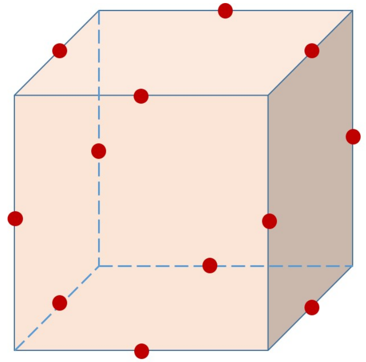
X cube model

Vijay, Haah, Fu (2016)

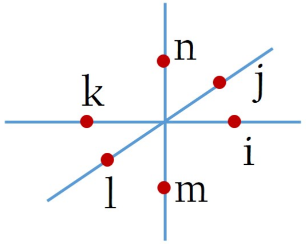
$$H_{XC} = -\sum_c A_c - \sum_{v,k} B_c^{(k)},$$

Exactly solvable `stabilizer' Hamiltonian in three dimensions. Sum of commuting projectors. Properties robust to small local perturbations.

$$H = -J\sum_c A_c - \sum_{v,k} B_c^{(k)} + \Lambda \sum_i \sigma_z + \lambda \sum_i \sigma_x,$$



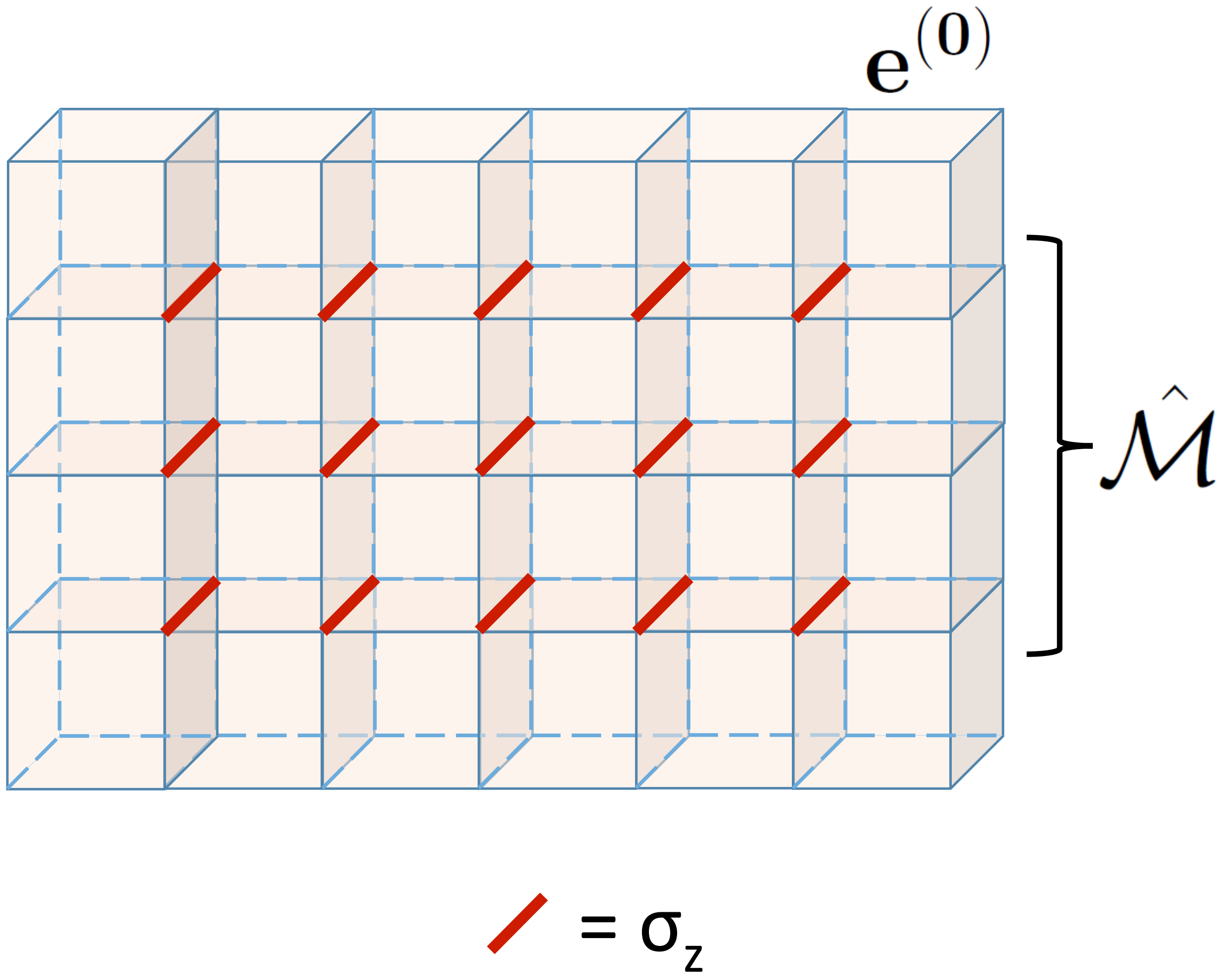
$$A_c = \prod_{j\in\partial c} \sigma_x^j$$



$$B_v^{(xy)} = \sigma_z^i \sigma_z^j \sigma_z^k \sigma_z^l$$
$$B_v^{(yz)} = \sigma_z^j \sigma_z^n \sigma_z^l \sigma_z^m$$
$$B_v^{(xz)} = \sigma_z^i \sigma_z^n \sigma_z^k \sigma_z^m$$

X-Cube Model: Excitations

- Fracton = -1 e-value of Cubic term
- No local operator can create single fractons.
- Isolated fractons created at ends of membrane operator.
- Cannot move fractons by acting with any local operator (without creating additional excitations)
- Totally immobile excitations

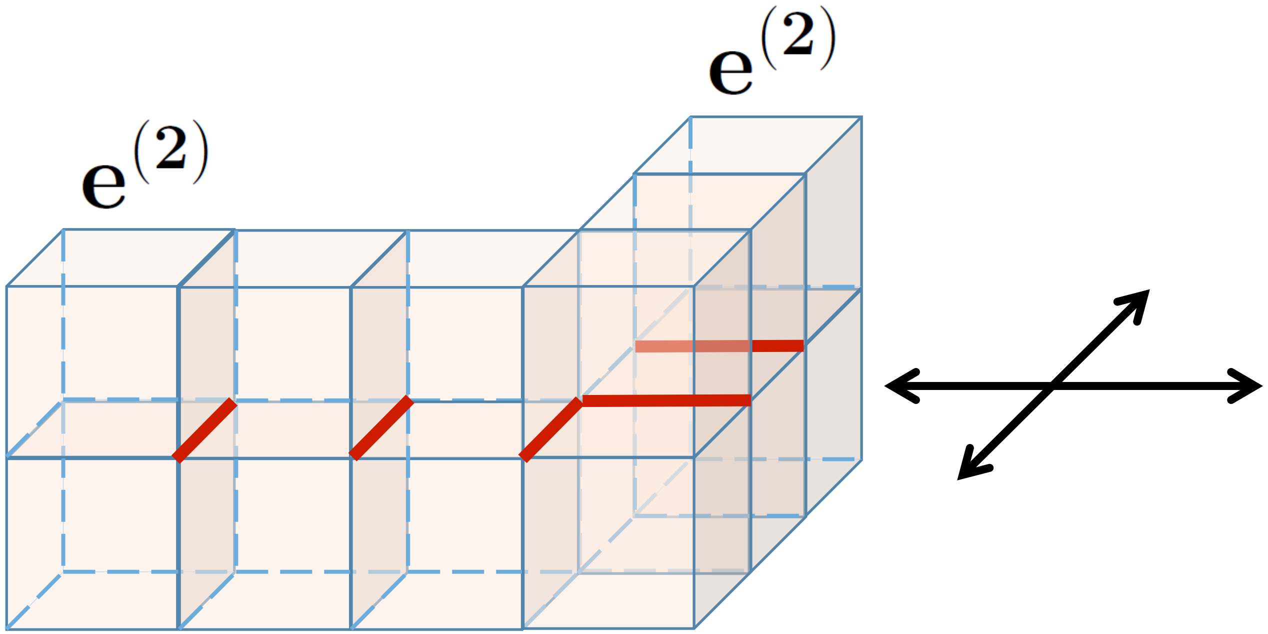


X-Cube Model: Excitations

Bound state of two fractons – created by Wilson line-like operator

– mobile in plane perp. to "stack"

Subdimensional excitations



Dynamics of fracton models

- Isolated fractons are *completely immobile*
- At zero energy density (subextensive number of excitations), the system retains forever a memory of its initial conditions under closed system Hamiltonian dynamics
- Many body localization in a translation invariant Hamiltonian! (but only at zero temperature) (Kim and Haah, 2015)

- What about finite energy density (extensive number of excitations?)

Type I Fractons at Finite Energy Density

Fractons, bosons, composites at temp. T << W

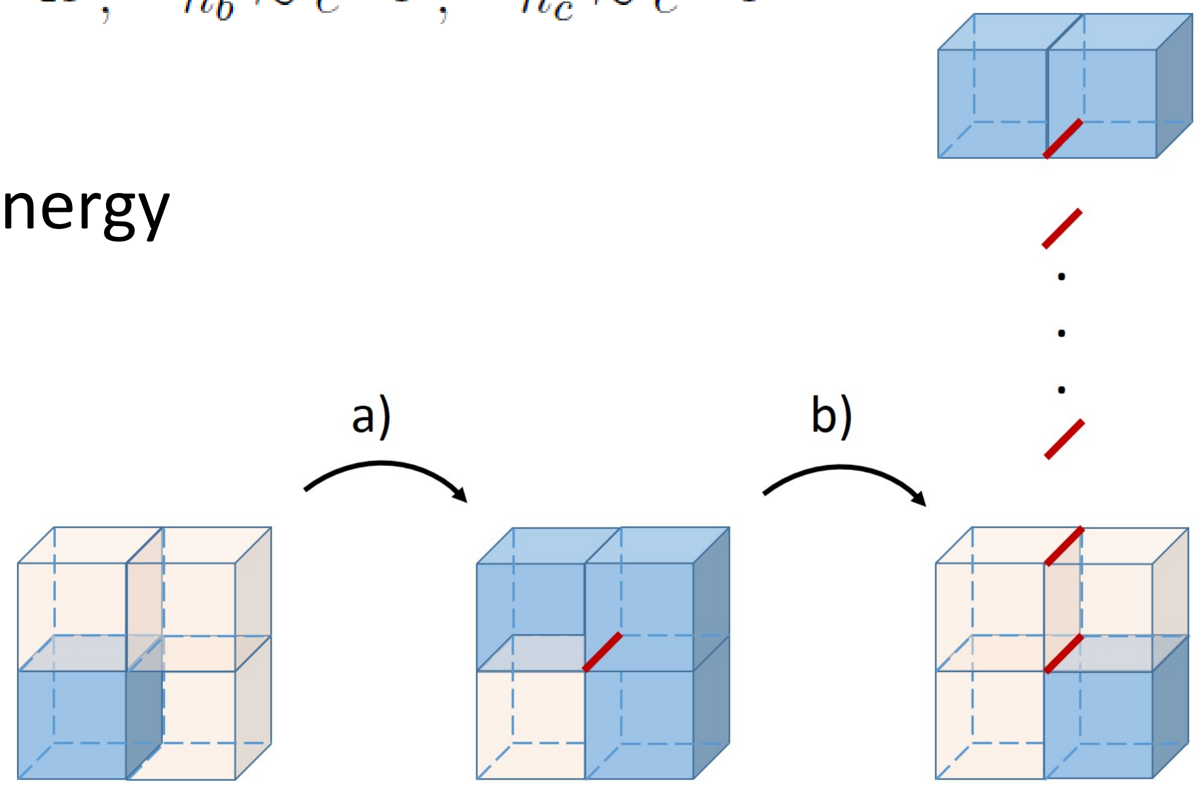
W = Charge gap (=4 in X-Cube)

Gapped → density fixed by T: $n_f \sim e^{-\frac{W}{2T}}, \quad n_b \sim e^{-\frac{W}{T}}, \quad n_c \sim e^{-\frac{2W}{T}}$

Single fracton hop takes system off energy shell by W.

Mobile composites act as

heat bath with bandwidth Λ << W



- Similar problems addressed in MBL literature (RN, Gopalakrishnan, Huse 2014; Gopalakrishnan and RN, 2014, RN and Gopalakrishnan, 2016)
- `Borrow' those analyses, apply to fractons

Usual case of activated transport:

$$\Gamma_A \sim n \sim e^{-\Delta/T}$$

Arrhenius relaxation.

Exponential slowness due to rarity of charge carriers.

Mobility typically O(1).

Fractons:

$$\Gamma \sim n_f \max\left(e^{-W/T}, e^{-W/\tilde{\Lambda}}\right) \sim n_f\, e^{-W/T}$$

"Asymptotic MBL"

Exponentially suppressed due to rarity of charge carriers.

Additional suppression due to mobility.

Equilibration b/w Fractons & Bath

Consider equilibration between fractonic and mobile sectors

Write down rate equations and solve.

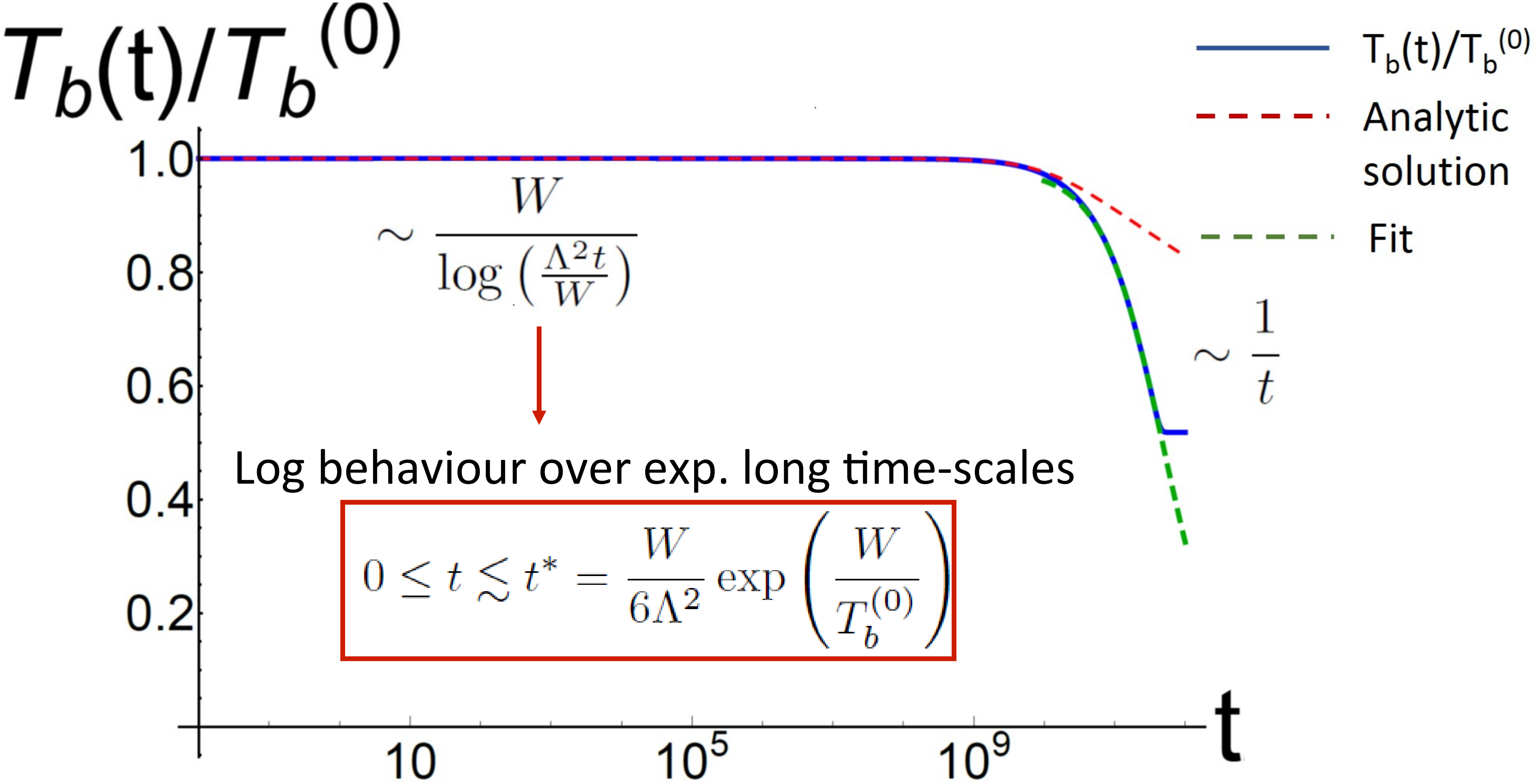
Charged sector (fractons), initial temp. $T_f^{(0)}$

Thermal sector (bosons & composites), initial temp. $T_b^{(0)} >> T_f^{(0)}$

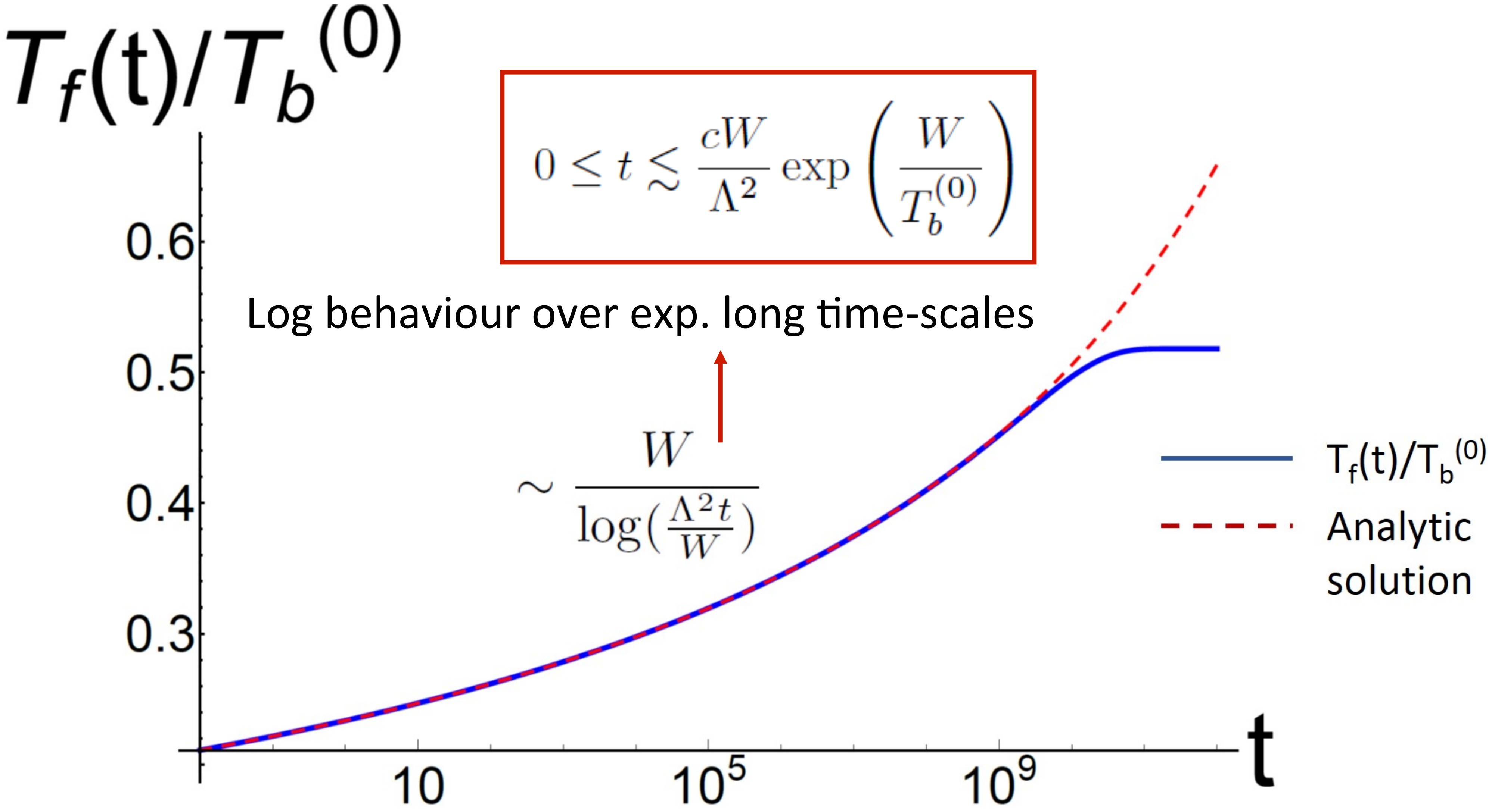
$$\frac{dT_b}{dt} = -\frac{\Lambda^2 T_b^2}{W^2}\left(3n_b + n_f - n_f^2 - \frac{n_f^3}{n_b} - 2\frac{n_f^4}{n_b}\right),$$

$$\frac{dT_f}{dt} = \frac{4\Lambda^2 T_f^2}{W^2}\left(3\frac{n_b^2}{n_f} + n_b - n_b n_f - n_f^2 - 2n_f^3\right).$$

Logarithmic Cooling of Bath



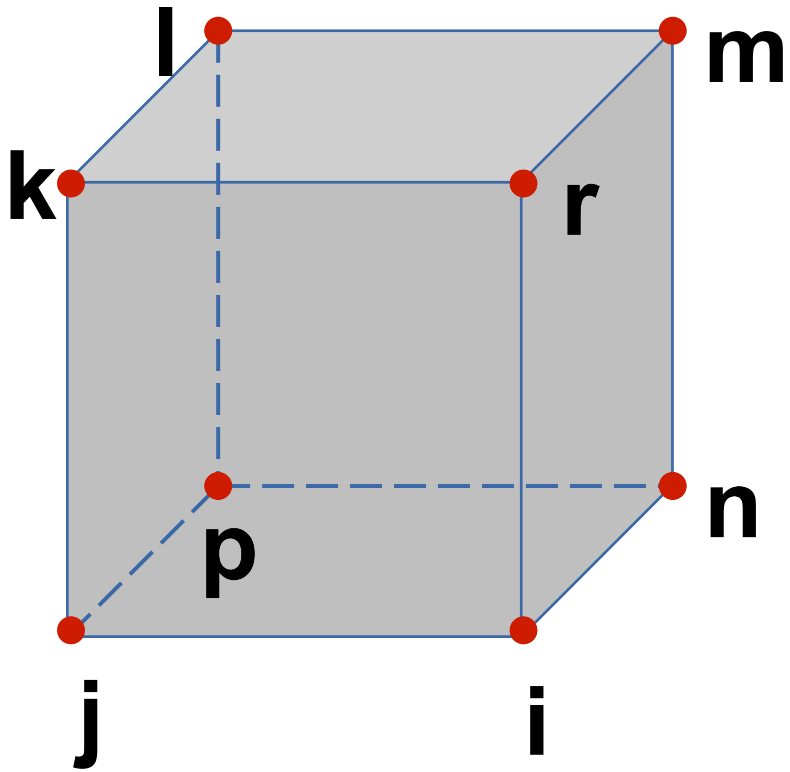
Logarithmic Heating of Fractons



Connections to classical glass

- `Kinetically constrained models' of classical glass have dynamical rules exhibiting `dynamical facilitation' (excitations can only move if next to other excitations)
- Such dynamical rules, imposed by hand, give glassy dynamics and logarithmic relaxation
- Here these dynamical rules emerge from Hamiltonian dynamics.

Haah's Code (Type II)



$$G_c^Z = \sigma_z^i \mu_z^j \sigma_z^k \mu_z^l \sigma_z^m \mu_z^n \sigma_z^r \mu_z^r$$

$$G_c^X = \sigma_x^i \mu_x^j \sigma_x^k \mu_x^l \sigma_x^m \mu_x^n \sigma_x^p \sigma_x^p$$

$$H = -J \sum_c G_c^X - \sum_c G_c^Z$$

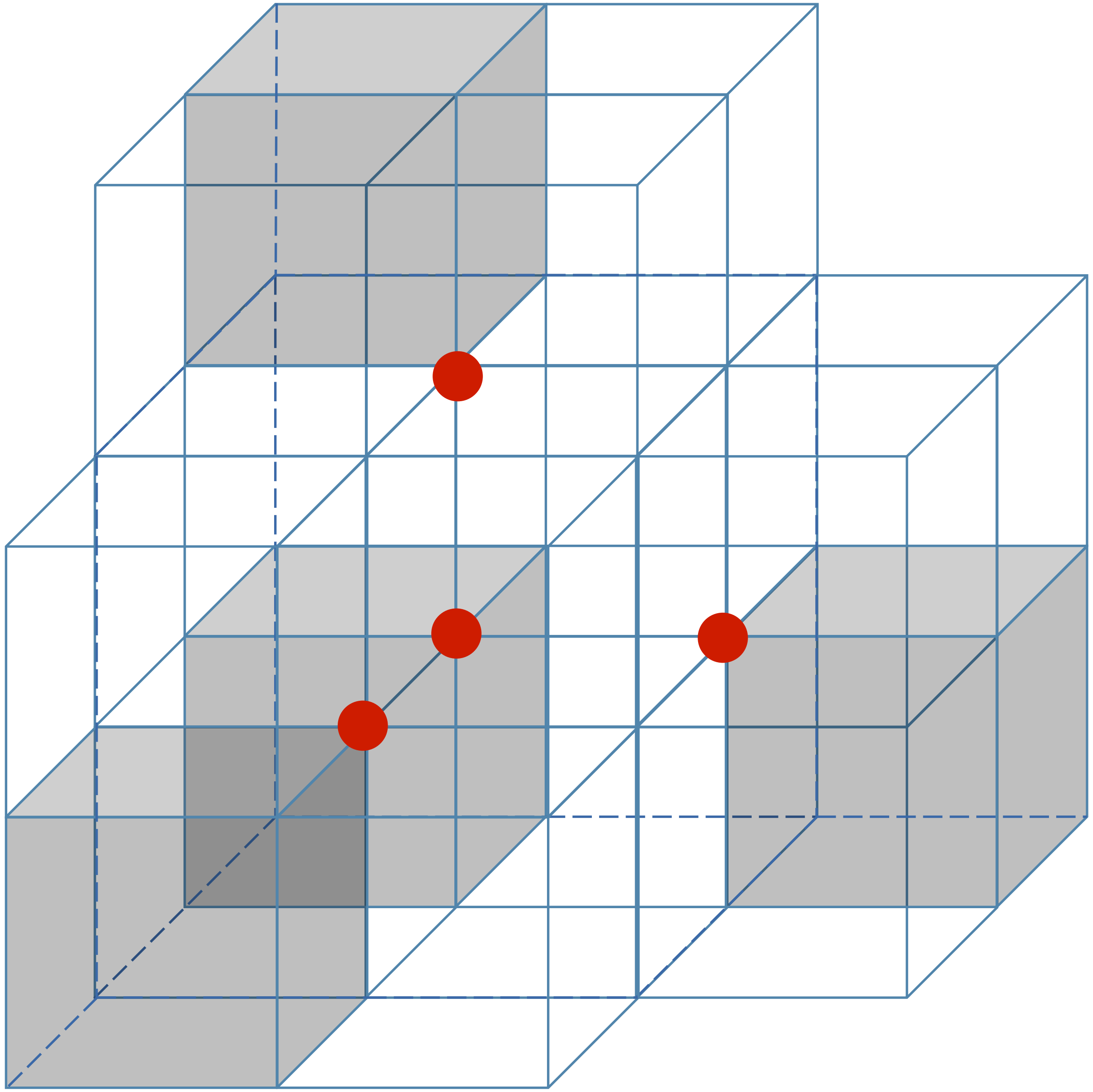
Defined on Cubic Lattice.

Two qubits on each vertex.

Sub-extensive log(GSD) on 3-torus (but diverging in the thermodynamic limit)

J. Haah (2011)

Haah's Code (Type II)



Move fractons by overlaying fractal operators. Each time spacing of fractons doubles

To move a distance R, overlay fractal operators log(R) times

If only local operators available, the maximum energy cost of intermediate states is proportional to the number of fractal overlays required – energy barrier to move fractons $\sim c \log(R)$

Bravyi &Haah (2011)

Sub-diffusion in type II models

Type II fracton system in contact w/ narrow bandwidth heat bath of composites at T

$$\Lambda \ll W$$

Initialise system with isolated fracton.

Logarithmic energy barrier: energy cost for moving distance $R = W\, c \log R$

Moving a distance R takes time

$$t = (R)^{\frac{cW}{\tilde{\Lambda}}} \; ; \quad \tilde{\Lambda} = \min(T, \Lambda)$$

$\tilde{\Lambda} \ll W$ Strong sub-diffusive behaviour in translation invariant 3D model

“Super-Arrhenius” scaling

System will eventually equilibrate : borrow energy from heat bath to create fractons in groups of 4.
Redistribute over system.

To achieve thermal density of fractons $n_f \sim \exp(-W/2T)$

Need to move fractons over length-scale $a \sim \exp(2W/3T)$

Equilibration time follows “super-Arrhenius” scaling

$$t_{equilibrate} \sim \exp\left(+c' \frac{W^2}{T^2}\right)$$

Fracton Dynamics Summary

- Fracton models are natural *translationally invariant* models exhibiting *glassy dynamics*
  - Approach to equilibrium is logarithmic in time
  - Fracton mobility is supressed (potentially super-exponentially)
  - Subdiffusion up to (potentially super-exponentially) long time

- Question of ongoing interest here e.g. U(1) variants of Haah's code may relax (preliminary) only on timescales *exp(exp(exp(1/T)))*

Thermodynamics of continuous fracton models

Prem, Pretko, Nandkishore, arXiv: 1709.09673

- Now consider correlated (ground state) physics of fractons at finite *charge* density.
- Need U(1) model to be able to control charge density independently.
- Introduce new language - `higher rank gauge theory’

Fractons as higher rank gauge theories

- Pretko (2016)
- Continuum U(1) gauge theory with symmetric tensor gauge fields
- Generalized Gauss Law constraint e.g. $\partial_i \partial_j E^{ij} = \rho,$
- Additional conservation law $\int \vec{x}\rho = \int x^k \partial_i \partial_j E^{ij} = -\int \partial_j E^{kj} = 0$

- Only processes that conserve dipole moment are allowed.
    - Charges are immobile (fractons). Cannot be created *or moved* by any local operator
- Lagrangians with symmetric tensor fields as test bed for fractons

Fractons at finite density

- Power law repulsion r^{-n} (different n for different theories)
- Attraction exp(-r) from kinetic energy (fractons can move only when close to other fractons, not when isolated)
- Competition of kinetic and potential energy gives rich phase diagram
  - $n>3$ `gravitational collapse’
  - $n<0$ `Wigner crystal’
  - $0<n<3$ `Micro-emulsions’ − clusters of finite size

Finite density of dipoles

- Conserved dipole moment means we have a second (*vector!*) chemical potential to play with.
- Turn on a finite density of dipoles and ask what happens.
- For now, consider only dipoles of a particular orientation.

Dipolar Fermi liquid

- Simplest higher rank theory in 3D has (angle dependent but always repulsive) 1/r interactions between fermionic dipoles
- Conjecture: Fermi liquid state (Friedel oscillations etc), also *screening*
- Test *fractons* of opposite charge have ~r attraction, which gets screened to log(r)
- Competition between log(r) screened attraction and entropy
- Temperature driven (KT like) transition whereby dipoles unbind into fractons - A finite temperature (Fracton) transition.

Fractonic quantum Hall states

- No fractonic stabilizer codes in d=2
- Higher rank gauge theories in d=2 unstable to Polyakov confinement
- This problem can be circumvented by adding a (higher rank) Chern Simons term (Pretko 2017) yielding fractonic theories in d=2
- These Chern-Simons fractonic theories may be understood (Prem-Pretko-Nandkishore) as states with a finite density of dipoles in 2D bulk, where dipoles are placed in *quantum Hall state* (integer or Laughlin)

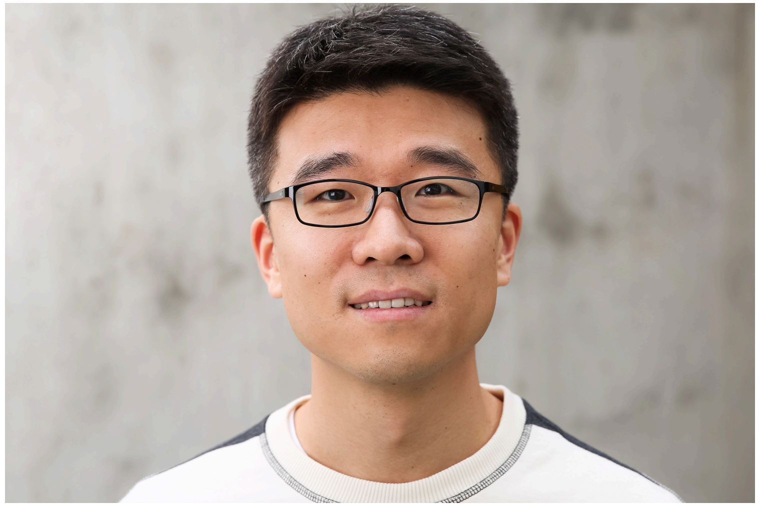
Fracton quantum Hall continued

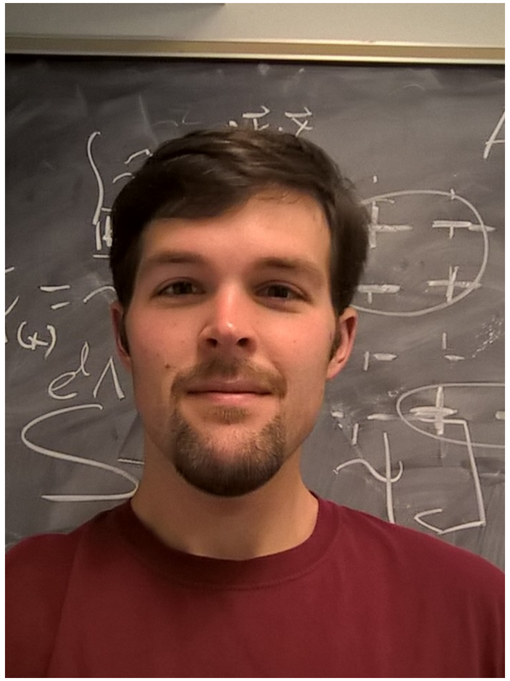
- (skipping all technical details)
- Quantum Hall states of *fractonic* charge (also thermal Hall effect)
- Can deduce edge theory, which is scale invariant 1D field theory (CFT?)
- Edge theory has gapless fracton excitations!
  - Towards fractons in 1D? Treated via CFTs instead of stabilizer codes? Work in progress.

Conclusions

- Fractons at finite energy density have rich *dynamics*
- Fractons at finite charge density (even at zero temperature) have rich *thermodynamic* structure.
- We have just scratched the surface − there is lots to be done!

Thanks to collaborators





FQXi

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
PRIMUM ACQUIRERE COGNITIONEM
UNITED STATES AIR FORCE

ALFRED P. SLOAN FOUNDATION
1934
Alfred P. Sloan
FOUNDATION